To illustrate how a $Z-\theta$ transmission line chart can be used to determine the transmission characteristics of a sandwich, an example will be considered. For this example, the polarization is perpendicular to the plane of incidence, $\lambda_0=3.2$ cm $=1.26$ inch, $\theta_0=45°$, $d_1=0.05$ inch, $\mu_1=1$, $\epsilon_1=6$, $d_2=0.21$ inch, $\mu_2=0.8(1+j0.05)$, $\epsilon_2=4(1-j0.2)$, $d_3=0.07$ inch, $\mu_3=1$, and $\epsilon_3=4$. Now $\eta_{0\perp}=\eta_{4\perp}=1.414$, $\eta_{1\perp}=0.426$, $\eta_{2\perp}=0.481\underline{/7.9°}$, $\eta_{3\perp}=0.535$, $\alpha_1=\alpha_3=0$, $\alpha_2=0.721$ nepers per inch, $\beta_1=(2\pi)(1.861)$ radians per inch, $\beta_2=(2\pi)\cdot(1.317)$ radians per inch, and $\beta_3=(2\pi)(1.485)$ radians per inch.

The point $Z_1=\eta_{0\perp}/\eta_{1\perp}=3.317$ is plotted on a Z-θ chart, as in Fig. 2 (below). The

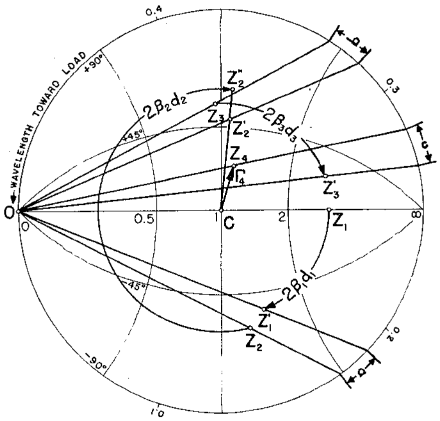

Fig. 2—Graphical construction for example.

point Z_1 is rotated through the angle $2\beta_1 d_1$ or $(1.861)(0.05)\lambda=0.093\lambda$ to obtain $Z_1'=1.40\underline{/-54°}$. The point $Z_2=(\eta_{1\perp}/\eta_{2\perp})Z_1'=1.24\underline{/-61.9°}$ is plotted. The point Z_2 is rotated through the angle $2\beta_2 d_2$ or $(1.317)\cdot(0.21)\lambda=0.278\lambda$ to obtain Z_2''. The distance $\overline{CZ_2''}e^{-2\alpha_2 d_2}=\overline{CZ_2''}e^{-0.0303}=0.739\overline{CZ_2''}$ is measured from C along the line CZ'' to locate the point $Z_2'=1.07\underline{/49°}$. The point $Z_3=(\eta_{2\perp}/\eta_{3\perp})Z_2'=0.96\underline{/56.9°}$ is plotted. The point Z_3 is rotated through the angle $2\beta_3 d_3$ or $(1.485)(0.07)\lambda=0.104\lambda$ to obtain $Z_3'=2.98\underline{/25°}$. The point $Z_4=(\eta_{3\perp}/\eta_{3\perp})Z_3'=1.13\underline{/25°}$ is plotted.

The magnitude of the voltage reflection coefficient is $\overline{CZ_4}$/(radius of chart) $=0.23$ Now

$$|E_4^+| = \frac{\overline{OC}}{\overline{OZ_1}}\frac{\overline{OZ_1'}}{\overline{OZ_2}}\frac{\overline{OZ_2'}}{\overline{OZ_3}}\frac{\overline{OZ_3'}}{\overline{OZ_4}}e^{\alpha_1 d_1+\alpha_2 d_2+\alpha_3 d_3} = 1.10$$

and the ratio of power transmitted to power incident is 0.826. The angle of E_4^+ is

$$\begin{aligned} a-b-c+\beta_1 d_1+\beta_2 d_2+\beta_3 d_3 &= 360°(0.014-0.016-0.016+0.093 \\ &\quad +0.278+0.104) \\ &= 164.5°, \end{aligned}$$

and the increase in phase retardation due to the sandwich is

$$\Phi=164.5°-(360°/1.26)(0.33)\cos 45°=9.78°.$$

A graphical construction for determining the transmission characteristics of a sandwich has the advantage that the effect of the individual layers is presented in visual form. Ways to vary the parameters to obtain a desired result may be suggested by a study of the chart.

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Measurement of Reflection Coefficients through a Lossless Network

Often it is not convenient to measure a reflection coefficient directly. In some such cases, the following procedure may be used. The arrangement of components shown in Fig. 1 will be considered. It will be assumed that lines Nos. 1 and 2 are lossless. These lines may be transmission lines, waveguides, or one of each.

Fig. 1—Arrangement for measuring reflection coefficients.

First, a matched load is connected to line No. 2 and the matching network is adjusted until the reflection coefficient in line No. 1 is zero. Next, the matched load is replaced by a short circuit. The magnitude of the reflection coefficient in line No. 1 should be one. This should be checked experimentally. A voltage null in line No. 1 is located and designated the *short-circuit point.* Finally, the short circuit is replaced by the unknown impedance. The reflection coefficient $\mathbf{\Gamma}_1$ in line No. 1 is measured relative to the short-circuit point. The reflection coefficient $\mathbf{\Gamma}_2$ in line No. 2 related to the point where the short circuit was connected is the same as $\mathbf{\Gamma}_1$.

The theoretical basis for this procedure is the well-known equation

$$\mathbf{\Gamma}_1=\frac{\boldsymbol{a}\mathbf{\Gamma}_2+\boldsymbol{b}}{\boldsymbol{c}\mathbf{\Gamma}_2+\boldsymbol{d}}.$$

Since $\mathbf{\Gamma}_1=0$ when $\mathbf{\Gamma}_2=0$, $\boldsymbol{b}=0$. Since the various components are lossless, $\Gamma_1=1$ when $\Gamma_2=1$. Consequently, $|\boldsymbol{c}\mathbf{\Gamma}_2+\boldsymbol{d}|$ is equal to a constant for all values of $\mathbf{\Gamma}_2$ such that $\Gamma_2=1$. This requires that $\boldsymbol{c}=0$. Now $\mathbf{\Gamma}_1=(\boldsymbol{a}/\boldsymbol{d})\mathbf{\Gamma}_2$, where $a/d=1$. When $\mathbf{\Gamma}_1$ is referred to the short-circuit point, $\boldsymbol{a}/\boldsymbol{d}=1$ and $\mathbf{\Gamma}_1=\mathbf{\Gamma}_2$.

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For a photograph and biography of S. B. Cohn, see the March, 1955 issue of the Transactions of the IRE-PGMTT.

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